

4d neutral dilatonic black holes and $(4 + p)$ dimensional nondilatonic black p -branes

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Abstract

It is shown that, in contrast to the case of extreme 4d dilatonic black holes, 4d neutral dilatonic black holes with horizon singularities can not be interpreted as nonsingular nondilatonic black p -branes in $(4 + p)$ dimensions, regardless of the number of extra dimensions p . That is, extra dimensions do not remove naked singularities of 4d neutral dilatonic black holes.

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It has been pointed out by Gibbons, Horowitz, and Townsend[1] that for certain values of the dilaton coupling $\alpha = \sqrt{p/(p+2)}$, where p is an odd integer, the horizon singularities of extreme 4d dilaton black holes[2] can be removed by extra dimensions. That is, for certain numbers of extra dimensions ($p = \text{odd}$) the extreme 4d dilaton black hole can be viewed from a $(4 + p)$ dimensional perspective as a singularity free nondilatonic black p -brane. It can be inferred that this result may not hold for the case of nonextreme dilaton black holes. In fact, Wesson and de Leon[3] have investigated

the neutral spherically symmetric 4d black hole solutions for 5d Kaluza-Klein theory and have shown that the horizon singularities of the neutral nonrotating dilatonic solutions are not removed by the extra dimension, *i.e.* the 5d nondilatonic neutral black 1-brane is not singularity free, except for the Schwarzschild case. This was done by computing the 5d Kretschmann invariant. The purpose of this paper is to extend this result to an arbitrary number p of extra dimensions. This can be accomplished by considering the neutral, spherically symmetric 4d dilaton solutions (in nonisotropic coordinates) of Brans-Dicke (BD) theory that arise from a reduction of the $(4 + p)$ dimensional action for pure gravity. In nonisotropic coordinates the Schwarzschild case is easily identified, and, without appealing to scalar no hair theorems, it can be seen that all other cases arise from singular black p -branes, regardless of the value of p .

In $(4 + p)$ dimensions the metric is given by

$$ds^2 = \tilde{g}_{MN} dX^M dX^N = g_{\mu\nu} dx^\mu dx^\nu - \delta_{mn} B^2(x) dy^m dy^n, \quad (1)$$

where $X^M = (x^\mu, y^m)$, with $M, N = 0, \dots, p+3$, $\mu, \nu = 0, \dots, 3$, $m, n = 1, \dots, p$, and the metric has a signature $\text{diag } g_{MN} = (+, -, \dots, -)$. The action for pure gravity is

$$S_{(4+p)} = \frac{1}{16\pi\bar{G}} \int d^4x d^p y \sqrt{\tilde{g}} \tilde{R}, \quad (2)$$

where \bar{G} is the gravitation constant for the $(4 + p)$ dimensional theory and $\tilde{g} = |\det(\tilde{g}_{MN})|$. We define the scalar field $\varphi = \frac{1}{\bar{G}} \left(\frac{B}{B_0}\right)^p$, where B_0 is a constant, $g = \det g_{\mu\nu}$, $R = g^{\mu\nu} R_{\mu\nu}$ is the 4d Ricci scalar, and the usual Newtonian gravitation constant G is related to \bar{G} by $\bar{G} = GV_p$, where the “volume” of the compactified internal space $V_p = B_0^p \int d^p y$ is assumed to be finite. The action then takes the form

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left\{ \varphi R + \frac{\omega}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right\}, \quad \omega = -1 + \frac{1}{p}. \quad (3)$$

This is just the action for Brans-Dicke theory in vacuum with a BD scalar field φ and a fixed BD parameter $\omega = -1 + \frac{1}{p}$. Therefore, vacuum solutions for the Brans-Dicke

theory corresponding to the particular value of BD parameter $\omega = -1 + \frac{1}{p}$ are also vacuum solutions for $(4+p)$ dimensional Einstein gravity, and these solutions can be interpreted as neutral nondilatonic black p -branes in $(4+p)$ dimensions.

The static spherically symmetric BD vacuum solutions [4],[5] have been studied previously by Campanelli and Lousto [6], Saa[7] Rama [8], and Kim [9]. Campanelli and Lousto (CL) have represented the solutions in the form

$$\begin{aligned} ds^2 &= A^{m+1} dt^2 - A^{n-1} dr^2 - r^2 A^n d\Omega^2, \\ \varphi &= \varphi_0 A^{-\frac{1}{2}(m+n)}, \quad A = \left(1 - \frac{2r_0}{r}\right), \end{aligned} \quad (4)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and φ_0 , r_0 , m , and n are constants. In terms of these solutions parametrized by m and n , the BD parameter is given by [6]

$$\omega = -2 \frac{(m^2 + n^2 + mn + m - n)}{(m+n)^2}. \quad (5)$$

The Schwarzschild solution is obtained for $m = n = 0$, in which case φ , and hence the extra dimensional scale factor B , is a constant. By examining the 4d Kretschmann curvature invariant $I = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$ Campanelli and Lousto have concluded that no physical singularity exists at the surface $r = 2r_0$ for solutions where either $n \leq -1$ or $m = n = 0$. For nonzero values of n where $n > -1$ the surface represents a naked singularity (in the BD frame defined by the metric and scalar field appearing in the BD action).

However, the $n \leq -1$ solutions of BD theory are not solutions of the $(4+p)$ dimensional pure gravity theory. To see this, we define $s = m + n$ and use the CL value of the BD parameter in (5), which can then be rewritten in the form

$$s = \frac{(n-1) \pm \sqrt{(n-1)^2 - 2n(n-2)(2+\omega)}}{(2+\omega)}. \quad (6)$$

For s to be real valued the quantity $f = (n-1)^2 - 2n(n-2)(2+\omega)$ must be nonnegative. For $-1 \leq \omega \leq 0$ we find that the constraint $f \geq 0$ excludes all values of $n \leq -1$. Therefore the $(4+p)$ dimensional Einstein theory does not give rise to

any of the 4d BD solutions for which $n \leq -1$. However, the Schwarzschild solution is allowed. We will make use of these facts shortly.

To determine whether the singularities of the 4d BD solutions at $r = 2r_0$ can be removed by the extra dimensions, we compute the $(4 + p)$ dimensional Kretschmann scalar $\tilde{I} = \tilde{R}_{ABMN} \tilde{R}^{ABMN}$ for the nondilatonic neutral black p -brane. Using the symmetry properties of the Riemann tensor, $\tilde{R}_{ABMN} = -\tilde{R}_{BAMN} = -\tilde{R}_{ABNM} = \tilde{R}_{MNAB}$, we can write $\tilde{I} = I + 4\tilde{R}^{\alpha b \mu n} \tilde{R}_{\alpha b \mu n} + \tilde{R}^{abmn} \tilde{R}_{abmn}$, where the four dimensional Kretschmann scalar $I = R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu}$ has been computed by CL and can be written in the form

$$I = \frac{4r_0^2}{r^6} A^{-(2n+2)} \left\{ \left(\frac{r_0}{r} \right)^2 J_1(m, n) + 4 \left(\frac{r_0}{r} \right) J_2(m, n) + 6J_3(m, n) \right\}, \quad (7)$$

where the $J_i(m, n)$ are polynomial functions of m and n and are given in [6]. A calculation for \tilde{I} then leads to the result

$$\tilde{I} = \frac{4r_0^2}{r^6} A^{-(2n+2)} \mathcal{B}(r; m, n), \quad (8)$$

where

$$\mathcal{B}(r; m, n) = \left\{ \left(\frac{2r_0}{r} \right)^2 K_1(m, n) + \left(\frac{2r_0}{r} \right) K_2(m, n) + K_3(m, n) \right\}, \quad (9)$$

with

$$\begin{aligned} K_1(m, n) &= 4pq^2 \{-4n + 4q + 4 \\ &\quad + \{(q + n)^2 - (q + n)(n - 1) + \left(\frac{p-1}{2}\right) q^2 \\ &\quad + \frac{1}{4}[(m + 1)^2 + (n - 1)^2 + 2n^2]\} \} + \frac{1}{4} J_1(m, n), \\ K_2(m, n) &= 4pq^2(4n + 4q - 10) + 2J_2(m, n), \\ K_3(m, n) &= 24pq^2 + 6J_3(m, n), \end{aligned} \quad (10)$$

and we have defined $q = \left(\frac{m+n}{2p}\right)$.

In order for \tilde{I} to be finite on the surface $r = 2r_0$, we must have that either (1) $n \leq -1$ or (2) $n > -1$ and $\mathcal{B}(2r_0; m, n) = 0$. As pointed out previously, the solutions satisfying the first condition are inaccessible to the $(4 + p)$ dimensional

Einstein theory, so that we must focus on solutions which satisfy the second condition, which is satisfied for $\mathcal{B}(r; m, n) \sim A^2$ with $n = 0$. This is equivalent to the conditions

$$K_2(m, 0) = -2K_1(m, 0), \quad K_3(m, 0) = K_1(m, 0). \quad (11)$$

Using the K_i in (10), we find that the only solution satisfying these conditions is the one for which $m = n = 0$, i.e. the Schwarzschild solution. Therefore, except for the Schwarzschild case, the neutral nonrotating solutions of the 4d dilatonic BD theory cannot be viewed as nonsingular nondilatonic p -branes in $(4+p)$ dimensions. Furthermore, the only nondilatonic black p -brane solutions for which the extra dimensions become visible[10] near $r = 2r_0$ are those possessing a singularity at $r = 2r_0$.

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